# AP Physics B Review Sheet

## UNITS OF MEASUEMENT

| | |
|---|---|
| distance / displacement | $m$ |
| mass | $kg$ |
| time | $s$ |
| speed / velocity | $m/s$ |
| acceleration | $m/s^2$ |
| force | $N=\frac{kg\cdot m}{s^2}$ |
| pressure | $Pa=\frac{N}{m^2}=\frac{kg}{m\cdot s^2}$ |
| impulse | $N\cdot s=\frac{kg\cdot m}{s}$ |
| momentum | $\frac{kg\cdot m}{s}$ |
| work / energy | $J=N\cdot m=\frac{kg\cdot m^2}{s^2}$ |
| power | $W=\frac{J}{s}=\frac{kg\cdot m^2}{s^3}$ |
| angular displacement | $rad$ |
| angular velocity | $rad/s$ |
| angular acceleration | $rad/s^2$ |
| torque | $N\cdot m$ |
| moment of inertia | $kg\cdot m^2$ |
| angular momentum | $kg\cdot m^2/s$ |
| rotational work/energy | $J$ |
| frequency | $Hz = cycles/s$ |
| mass density | $kg/m^3$ |
| temperature | $K$ or $°C$ |
| heat | $J$ |
| entropy | $J/K$ |
| electric charge | $C$ |
| electric field | $N/C$ |
| electric flux | $N\cdot m^2/C$ |
| electric potential | $V=J/C$ |
| potential gradient | $V/m$ |
| capacitance | $F=C/V$ |
| electric current | $A=C/s$ |
| electrical resistance | $\Omega=V/A$ |
| resistivity | $\Omega\cdot m$ |
| magnetic field | $T=N\cdot s/C\cdot m$ |
| magnetic flux | $Wb=T\cdot m^2$ |
| inductance | $H=V\cdot s/A$ |

To convert units, make sure you write down units explicitly and treat them as algebraic quantities. You will multiply by conversion factors in the form of fractions so units cancel.

## TRIGONOMETRY

$$\sin\theta=\frac{opp}{hyp} \qquad \theta=\sin^{-1}\left(\frac{opp}{hyp}\right)$$

$$\cos\theta=\frac{adj}{hyp} \qquad \theta=\cos^{-1}\left(\frac{adj}{hyp}\right)$$

$$\tan\theta=\frac{opp}{adj} \qquad \theta=\tan^{-1}\left(\frac{opp}{adj}\right)$$

$$\sin^2\theta+\cos^2\theta=1$$

## KINEMATICS IN ONE DIMENSION

$$\vec{v}_{avg}=\frac{\Delta\vec{x}}{\Delta t} \qquad v_{instant}=\lim_{t\to 0}\frac{\Delta\vec{x}}{\Delta t}$$

$$\vec{a}_{avg}=\frac{\Delta\vec{v}}{\Delta t} \qquad a_{instant}=\lim_{t\to 0}\frac{\Delta v}{\Delta t}$$

| Variables | Equations |
|---|---|
| $x_0$ | $v=v_0+at$ |
| $x$ | $x=x_0+v_0t+\frac{1}{2}at^2$ |
| $v_0$ | |
| $v$ | $v^2=v_0^2+2a\Delta x$ |
| $a$ | |
| $t$ | |

1. Make a drawing to represent the situation
2. Decide which directions are positive relative to a conveniently chosen coordinate system
3. Write down the values of the kinematic variables. If there are multiple objects, remember that they may share come common variables.
4. Choose the equation which contains the given values and the unknown value.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next segment.
6. Remember that there may be two possible mathematical solutions, and you need to select the appropriate answer or answers when this occurs.

**Free Fall**

The acceleration due to gravity at sea-level at the Earth's equator is called one "gee" and is approximately 9.80 m/s$^2$.

## KINEMATICS IN TWO DIMENSIONS

Separate vectors into components parallel to the axes of the chosen coordinate system.

$$opp=hyp\sin\theta \qquad adj=hyp\cos\theta$$

Motion in the x-direction is independent of motion in the y-direction.

**Projectile Motion**

$$a_x=0\,m/s^2 \qquad v_x=v_{0x}$$

$$a_y=9.80\,m/s^2\text{ down}$$

At the apex of the flight (vertex of parabola)

$$v_y=0\,m/s$$

At original height on way back down

$$y=y_0 \qquad v_y=-v_{0y}$$

**Relative Velocity**

$v_{AB}$ is the velocity of A as seen by B

$$v_{AB}=v_{AC}+v_{CB} \qquad v_{AB}=-v_{BA}$$

## DYNAMICS (FORCES)

*Inertia* is the natural tendency of an object to remain at rest or in motion at a constant velocity. *Mass* is the quantitative measurement of inertia.

An *inertial reference frame* is one in which Newton's First Law of Motion is valid. The acceleration of any inertial reference frame is always zero.

An object is in *equilibrium* when it has zero acceleration.

**Newton's First Law of Motion**

An object continues in a state of rest or a state of motion at a constant velocity (speed and direction) unless compelled to change that state by a net external force.

**Newton's Second Law of Motion**

$$\sum\vec{F}=m\cdot\vec{a}$$

The direction of the acceleration is the same as the direction of the net force.

**Newton's Third Law of Motion**

Whenever a body exerts a force on a second body, the second body will exert an oppositely directed force of equal magnitude on the first body.

This force pair doesn't cancel because they are acting on different objects.

**Weight**

The force of gravity, $F_g$, acting on an object is often called the object's *weight*. If the local value of the acceleration due to gravity is known, this is $\vec{F}_g=m\cdot\vec{g}$

**Normal Force**

When an object is in contact with a surface, the *normal force*, $F_N$, is the component of the force that the surface exerts that is perpendicular to the surface; it is the force that prevents the object from passing through the surface.

**Apparent Weight**

The *apparent weight* of an object is the force that a scale exerts on the object resting on it, $F_N$. If the reference frame has an acceleration $a$ where up is positive, then $\vec{F}_N=\vec{F}_g+m\cdot\vec{a}$

**Tension**

*Tension* is a force applied to one end of a rope or cable that is transmitted to an object attached to the other end of the rope or cable.

If the rope is massless, the force applied to one end would be completely transmitted to the object at the other end. However, ropes do have mass, so some of the force applied is needed to accelerate the rope, which results is a reduced force acting on the attached object.

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**Friction**
When an object is in contact with a surface, the *friction*, $f$, is the component of the force that the surface exerts that is parallel to the surface; it is present only when the object is moving or attempting to move along the surface due to some other force acting on the object.

When the object is stationary, the magnitude of the *static frictional force* is $f_s$, which is only large enough to prevent motion up to some maximum amount $f_s^{max}=\mu_s \cdot F_N$ $\mu_s$ is the *coefficient of static friction*, and measures the roughness of the surface and object.

When the object is moving, the magnitude of the *kinetic frictional force* is $f_k$, which is given by $f_k=\mu_k \cdot F_N$ $\mu_k$ is the *coefficient of kinetic friction*, and measures the roughness of the surface and object. $\mu_k<\mu_s$ but either may be larger than 1 (normally not)

**Pressure**
*Pressure*, $P$, is the force exerted per unit area.

$$P=\frac{F}{A}$$

## IMPULSE AND MOMENTUM

The *impulse* of a force is given by $\vec{F}\cdot\Delta t$
An object's *linear momentum* is $\vec{p}=m\cdot\vec{v}$
Momentum is thought of as *inertia in motion.*

**Impulse-Momentum Theorem**
When a net force acts on an object, the impulse of the net force is equal to the change in the momentum of the object.

$$\vec{F}\cdot\Delta t=\Delta\vec{p}$$

If the mass remains constant while the net force is acting, this becomes

$$\vec{F}\cdot t=m\cdot\vec{v}_f-m\cdot\vec{v}_i$$

**Conservation of Linear Momentum**
The total linear momentum of an isolated system remains constant. An *isolated system* is a system for which the vector sum of the external forces acting on it is zero.

**Center of Gravity / Center of Mass**
The *center of gravity* is the point that represents the average location for the total weight of the system; it is the balance point for the object. The center of gravity of a thrown, rotating object is the point that moves along the parabola.

The *center of mass* is the point that represents the average location for the total mass of the system. The center of mass is generally the same point as the center of gravity, unless the object is tall enough that $g$ is smaller at its top than at its base, in which case the center of mass is slightly higher than the center of gravity.

## WORK AND ENERGY

The *work* done on an object of mass $m$ by a constant force $F$ is $W=(F\cos\theta)\cdot s$ where $\theta$ is the angle between the force and the displacement. Work is a scalar.

*Power* is the rate at which work is done.

$$P=\frac{W}{t}=F\cdot v$$

The *kinetic energy* of an object with mass $m$ and speed $v$ is $KE=\frac{1}{2}m\cdot v^2$

The *gravitational potential energy* of an object with mass $m$ at a height $h$ above a conveniently chosen zero point is $PE=m\cdot g\cdot h$

The *mechanical energy* is $E=KE+PE$

A *conservative force* is a force where the work done to move an object is independent of the path taken between the starting and ending point. Alternatively, a force is conservative when the work done in moving an object around a closed path is zero.

**Work-Energy Theorem**
When a net external force does work on an object, the work done is equal to the change in the kinetic energy of the object.

$$W=\Delta KE$$

The net work done by all nonconservative forces is equal to the change in the mechanical energy of the object.

$$W_{nc}=\Delta E=\Delta KE+\Delta PE$$

**Conservation of Mechanical Energy**
The total mechanical energy of an object remains constant as the object moves, provided that the net work done by any external nonconservative forces is zero.

**Conservation of Energy**
Energy can neither be created nor destroyed, but can only be converted from one form to another.

## UNIFORM CIRCULAR MOTION

An object is in *uniform circular motion* when it is traveling at a constant speed on a circular path.

An object spinning around an internal axis is *rotating*. An object spinning around an external axis is *revolving*.

The *period, T,* is the time it takes to travel once around the circle.

The linear speed, $v$, around the circular path is

$$v=\frac{2\pi r}{T}$$

**Centripetal Acceleration**
Since an acceleration is the rate of change in the velocity, which includes direction, an object in U.C.M. is accelerating. This is called its *centripetal acceleration*.

$$a_c=\frac{v^2}{r}$$

Centripetal acceleration always points toward the center of the circle, since that is the direction the centripetal force points.

**Centripetal Force**
The force that keeps the object moving along the circular path is called the *centripetal force*.

$$F_c=m\cdot a_c=\frac{mv^2}{r}$$

The centripetal force must point toward the center of the circle in order to force the object along the circular path. It is always perpendicular to the direction of motion.

A very common trick to solving U.C.M. Problems is to equate the force providing the centripetal force with the centripetal force formula.

**Maximum speed around an unbanked curve**

$$v=\sqrt{\mu_s g r}$$

**Speed around a frictionless banked curve**

$$\tan\theta=\frac{v^2}{rg}$$

**Orbital speed of satellites in circular orbits**

$$v=\sqrt{\frac{GM_E}{r}}$$

Note that $r=r_E+h$ is the orbital radius.

**Artificial Gravity**

$$v=\sqrt{r g_{effective}}$$

# AP Physics B Review Sheet

## ROTATIONAL KINEMATICS

Rotational kinematics problems are solved like linear motion problems, with angular variables substituted for linear variables.

| | Linear (m) | Angular (radians) |
|---|---|---|
| Distance | $d$ | $\theta = \frac{arclength}{radius}$ |
| Speed | $v = \frac{2\pi r}{T}$ | $\omega = \frac{\Delta\theta}{\Delta t} = \frac{1}{T}$ |
| Acceleration | $a$ | $\alpha = \frac{\Delta\omega}{\Delta t}$ |

| Variables | Kinematics Equations |
|---|---|
| $\theta_0$ | $\omega = \omega_0 + \alpha t$ |
| $\theta$ | $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ |
| $\omega_0$ | $\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$ |
| $\omega$ | |
| $\alpha$ | |
| $t$ | |

As long as the angular variables are expressed using radian measure, the following conversions can be made:

$$s = r\theta \quad v_T = r\omega \quad a_T = r\alpha$$

where the linear measure is tangential to the circular path. ($s$ is the arc length)

### Centripetal Acceleration with Angular Speed

$$a_c = \frac{v_T^2}{r} = r\omega^2$$

## ROTATIONAL DYNAMICS

### Torque

When a force $\vec{F}$ acts at a point a displacement $\vec{r}$ from the axis of rotation, it produces a *torque*

$$\vec{\tau} = \vec{r} \times \vec{F} = F \cdot r \cdot \sin\theta = F_{perpendicular} \cdot r = F \cdot l$$

$$F_{perpendicular} = F\sin\theta$$

$$l = r\sin\theta$$

The *lever arm, l,* is the distance between the line of action for the force and the axis of rotation for the object.

If the object does not have an axis of rotation fixed by an external object, the axis of rotation will be through the object's center of gravity.

### Equilibrium

For an object to be in equilibrium, the net force and the net torque acting on it must both be zero.

### Moment of Inertia

The equivalent of mass for a rotating object is the *moment of inertia, I.* It is calculated by

$$I = \sum_i m_i \cdot r_i^2 = \int_0^M r^2\,dm$$

### Common Moments of Inertia

Many more can be found on the Internet

| | |
|---|---|
| Point mass | $I = mr^2$ |
| Hollow cylinder, hoop | $I = mr^2$ |
| Solid cylinder, disk | $I = \frac{1}{2}mr^2$ |
| Thin rod around center | $I = \frac{1}{12}mr^2$ |
| Thin rod around end | $I = \frac{1}{3}mr^2$ |
| Hollow sphere around center | $I = \frac{2}{3}mr^2$ |
| Solid sphere around center | $I = \frac{2}{5}mr^2$ |

### Newton's Second Law for Rotation

$$\sum \vec{\tau} = I\vec{\alpha}$$

### Angular Momentum

$$\vec{L} = I\vec{\omega}$$

Total angular momentum is conserved if the net external torque acting on the system is zero.

Conservation of angular momentum is why an ice skater rotates faster when she pulls her arms and legs in during a spin.

### Rotational Kinetic Energy

$$KE_{rot} = \frac{1}{2}I\omega^2$$

Rotational kinetic energy is part of mechanical energy, and so must be part of the conservation of the total mechanical energy, though not conserved itself.

### Work-Energy Theorem for Rotation

$$\vec{\tau}_{net} \cdot \theta = \Delta\left(\frac{1}{2}I\omega^2\right)$$

### Power for Rotational Work

$$P = \tau\,\omega$$

## UNIVERSAL GRAVITATION

### Kepler's First Law of Planetary Motion

The paths of the planets are ellipses with the Sun at one focus.

### Kepler's Second Law of Planetary Motion

An imaginary line from the sun to a planet sweeps out equal areas in equal time intervals.

This implies that the planet moves faster when close to the sun (near *perihelion*) and slower when further from the sun (near *aphelion*).

We often use the conservation of mechanical energy instead of this law to determine speeds at different points in an elliptical orbit.

### Kepler's Third Law of Planetary Motion

For two objects orbiting the same body.

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

For one object orbiting a body with known mass $M$

$$T^2 = \frac{4\pi^2}{GM}r^3$$

### Newton's Law of Universal Gravitation

$$F_g = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67428 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$$

When using $G$, you must use meters for distance, kilograms for mass, and seconds for time.

### Orbital speed of satellites in circular orbits

$$v = \sqrt{\frac{GM}{r}}$$

Note that $r = r_E + h$ is the orbital radius.

### Escape speed at distance *d*

$$v = \sqrt{\frac{2GM}{d}}$$

### Local Acceleration Due to Gravity and Gravitational Field Strength

Outside a planet g is given by

$$g = \frac{GM}{r^2}$$

where $r = r_{planet} + h$ is the orbital radius.

Inside a planet with uniform density it is

$$g = \left(\frac{r_{current}}{r_{planet}}\right) g_{surface}$$

### Ocean Tides

Generally there are two high tides and two low tides each day, caused by the difference in the gravitational pull by the Moon on opposite sides of the Earth.

$$F_{tide} = \frac{4GM_{cause}R_{experiencing}}{d^3_{\text{center to center}}}$$

The Sun also causes tides, about half that of the Moon. When the Sun's tides and Moon's tides align (at the full moon and new moon) it is a *spring tide.* When they are 90° out of alignment (at the quarter moons) it is a *neap tide.*

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## SIMPLE HARMONIC MOTION

A *vibration* is a wiggle in time.
A *wave* is a wiggle in space and time and carries energy.

The *equilibrium position* is where the net force on the object is zero.

In order for an object to be in *simple harmonic motion*, the restoring force (the force that tries to return the object to its equilibrium position) must be proportional to the displacement from equilibrium.

The *amplitude*, *A*, of the motion is the maximum distance that the object moves away from equilibrium.

The *period*, *T*, is the time needed for an object to repeat one complete cycle of the motion.

The *frequency* of vibration, *f*, which is the number of cycles that repeat in one time period.

$$f=\frac{1}{T}$$

### Springs - Hooke's Law

The restoring force provided by a spring with *spring constant k* is

$$F_{spring}=-k\cdot x$$

when *x* is the displacement from the equilibrium position.

The spring constant is referred to as the stiffness of the spring, and is inversely proportional to the number of coils in the spring

The *period* of the spring is given by

$$T=2\pi\sqrt{\frac{m}{k}}$$

### Potential Energy of Springs

$$PE_{\text{spring}}=\frac{1}{2}k\,x^2$$

### Pendulums

The *period* of a pendulum is given by

$$T=2\pi\sqrt{\frac{L}{g}}$$

### Resonance

A *natural frequency* of an object is one at which minimum energy is required to produce forced vibrations. It is also the frequency that requires the least amount of energy to continue the vibration.

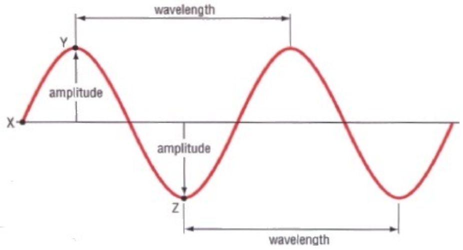
An object's natural frequencies depend on factors such as its elasticity and the shape of the object.

When the frequency of the application of a force to an object matches the object's natural frequency, a dramatic increase in amplitude occurs. This increased amplitude of the vibrations is called *resonance*.

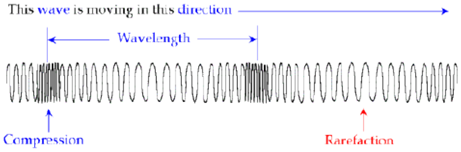
## WAVE MOTION

### Types of Waves

A *transverse wave* is created when the vibration that creates the wave is perpendicular to the direction of the wave. (light)



A *longitudinal wave* is created when the vibration that creates the wave is parallel to the direction of the wave. (sound)



*Water waves* are an example of waves that involve a combination of both longitudinal and transverse motions. As a wave travels through the water, the particles travel in circles. The radius of the circles decreases as the depth into the water increases.

### Parts of Waves

The high points are called *crests*.

The low points are called *troughs*.

The solid dark center line represents the *midpoint* of the vibration, where there is no restoring force acting on the object.

The *amplitude, A,* is the displacement from the midpoint vibration.

The *wavelength, λ,* of the wave is the distance between successive identical parts of the wave.

The *period, T,* of the wave is how long it takes for one wavelength to pass a fixed location.

The *frequency, f,* of the wave is how many waves pass occur in a given time.

### Wave Speed

$$v=\lambda f$$

On a string: $v=\sqrt{\frac{F_{tension}}{m/L}}$

## REFLECTION

### Wave Reflection in 1-Dimension

Waves encountering a hard boundary will flip (crest becomes trough). Waves encountering a soft boundary will reflect the way they come in (crest stays crest).

Waves moving from a less dense to a more dense medium reflect as if encountering a hard boundary (less amplitude) and transmit a wave with less amplitude and speed.

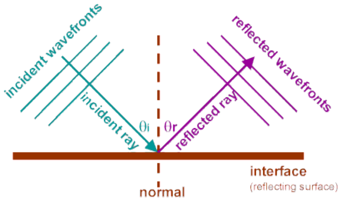
Waves moving from a more dense to a less dense medium reflect as if encountering a soft boundary (less amplitude) and transmit a wave with greater amplitude and speed.

### Wave Reflection in 2-Dimensions

A *wave front* is a line that represents the crest of a wave in two dimensions, and can be used to show waves of any shape.

*Rays* are lines that are perpendicular to the wave fronts and point in the direction of the velocity of the wave.

When parallel wave fronts strike a solid boundary, they reflect.



$$\theta_r=\theta_i$$

*Specular reflection* occurs if all of the reflected rays are parallel to each other which creates a sharp image.

*Diffuse reflection* occurs if the reflected rays are not parallel to each other which creates a fuzzy image or no image at all.

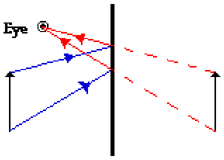
### Images

A *real image* is formed when the rays of light actually emanate from the image. A *virtual image* is formed when the rays of light appear to come from the image, but do not.

An image is *upright* if it is in the same orientation as the object that formed it. An image is *inverted* if it is in the opposite orientation.

### Plane Mirrors

A plane mirror forms an upright, virtual image the same size as the object that is located as far behind the mirror as the object is in front of it.



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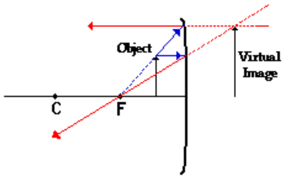
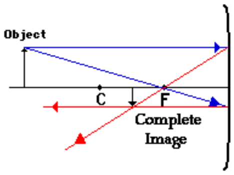
### Concave Mirrors

A concave mirror will have a focal point in front of the mirror. The mirror curves away from the object.

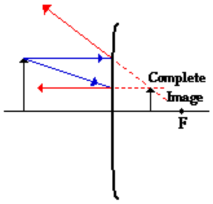
Concave mirrors form an upright, enlarged, virtual image if the object is closer than the focal point.



They will form an inverted, enlarged, real image if the object is between the focal point and the center of curvature.



They will form an upright, reduced, real image if the object is further than the center of curvature.



### Convex Mirrors

Convex mirrors curve toward the object, and have their focal point behind the mirror.

They will always form an upright, reduced, virtual image.

### Ray Tracing

Ray 1 is initially parallel to the principal axis to the mirror, then through the focal point.
Ray 2 is through the focal point to mirror, then parallel to the principal axis.
Ray 3 can be drawn to confirm the point of intersection, it is though the center of curvature.

### Mirror Equations

*f* is the focal length
- + for a concave mirror
- – for a concave mirror

$d_o$ is the object distance, $d_i$ is the image distance
- + in front of mirror (real)
- – behind mirror (virtual)

*m* is the magnification
- + for upright
- – for inverted

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o}$$

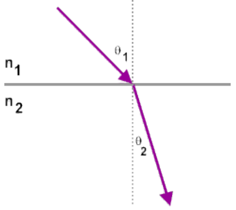
### REFRACTION

When parallel wave fronts strike a soft boundary (like that caused by changing the depth of water), the waves refract or change direction.

*Refraction* is the bending of a wave resulting from a change in its velocity as its moves from one medium to another. Since the frequency of a wave cannot change, independent of the source changing its frequency when it originally emits a wave, this change in wave velocity must result from a change in its wavelength in the second medium.

### Snell's Law

The *index of refraction* of a material is the ratio of the speed of light in a vacuum to the speed of light in the material. It must be at least 1.



$$n=\frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

### Total Internal Reflection

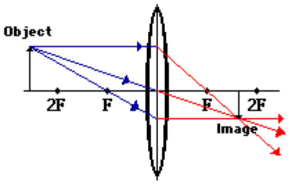
When light encounters a boundary where $n_1 > n_2$ then it is possible that all light reflects and none refracts through the boundary. This will happen at angles of incidence greater than the *critical angle* given by

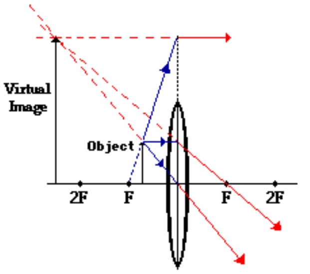
$$\sin\theta_c=\frac{n_2}{n_1}$$

### Converging Lenses

Convex lenses are *converging lenses* in that they focus light rays closer together.

Outside *f*



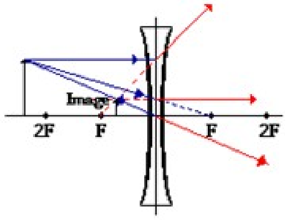


Inside *f*

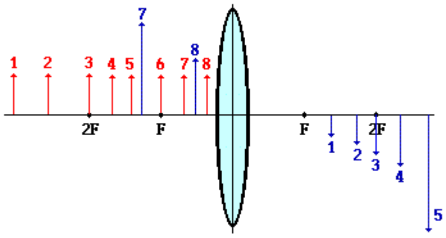
At *f* no image if formed because the rays are parallel.

### Diverging Lenses

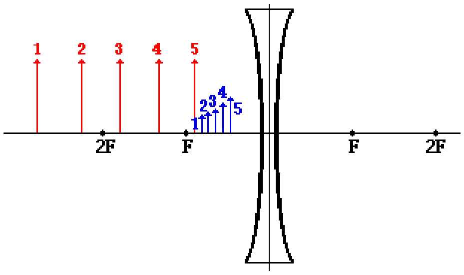
Concave lenses are *diverging lenses* in that they spread the light rays further apart.



### Object-Image Relations for Lenses



Each of the numbered objects (except #6) has an image with the corresponding number; its relative location, size, and orientation are shown.



### Ray Tracing

Ray 1 is initially parallel to the principal axis to the lens, then through the focal point for the first lens surface.
Ray 2 is through the focal point to second surface of the lens, then parallel to the principal axis.
Ray 3 can be drawn to confirm the point of intersection, it is though the center of the lens.

### Lens Equations

*f* is the focal length
- + for a converging lens
- – for a diverging lens

$d_o$ is the object distance
- + in front of lens (real)
- – behind lens (virtual)

$d_i$ is the image distance
- + behind lens (real)
- – in front of lens (virtual)

*m* is the magnification
- + for upright
- – for inverted

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

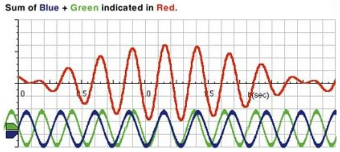
$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o}$$

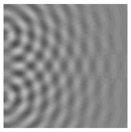
# AP Physics B Review Sheet

## INTERFERENCE AND DIFFRACTION

### Wave Interference

Overlapping waves add together by adding the amplitudes of the waves at every point.





*Constructive interference* occurs where the amplitude of the combined wave is greater than the amplitudes of the component waves.

*Destructive interference* occurs where the amplitude of the combined wave is less than the amplitude of the component wave (because one is above the midpoint and one is below the midpoint).

For two wave sources vibrating in phase, a difference in path length of zero, or an integral number of wavelengths leads to constructive interference. A difference in path length that is a half-integer number (0.5, 1.5, 2.5, etc.) leads to destructive interference.

### Young's Double Slit Experiment

When a parallel wave with wavelength $\lambda$ passes through two small slits separated by a distance $d$ then there are regions of maximum intensity found at angles given by

$$\sin\theta = \frac{n\lambda}{d}$$

and regions of minimum intensity at angles given by

$$\sin\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{d}$$

### Diffraction

The bending of a wave around an obstacle or the edges of an opening is called *diffraction*.

When a wave with wavelength $\lambda$ passes through a slit of width $d$ then there are regions of maximum intensity found at angles given by

$$\sin\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{d}$$

and regions of minimum intensity at angles given by

$$\sin\theta = \frac{n\lambda}{d}$$

### Resolving Power

Two point objects are just resolved as separate when the first dark fringe in the diffraction pattern of one falls directly on the central bright fringe in the diffraction pattern of the other.

$$\theta_{min} \approx 1.22\frac{\lambda}{d}$$

### Beats

When two overlapping waves have frequencies that are only slightly different, they create a combined wave with a *beat frequency* equal to the difference in the original frequencies.

### Standing Waves

A *traveling wave* obviously advances, and moves forward.

A *standing wave* appears to vibrate in place. The parts of the standing wave that appear stationary are called *nodes*. The positions of the standing wave with the greatest amplitude are known as *antinodes*. Antinodes appear halfway between nodes.

Standing waves are the result of interference. When two waves with equal amplitude pass through each other in opposite directions, the waves are always out-of-phase at the nodes, and in-phase at the anti-nodes.

A variety of standing waves can be produced by varying the frequency of vibration. Different standing waves are called *modes*. In musical instruments, the different vibrational modes result in different *harmonics* and *overtones*.

### Natural Frequency

The *natural frequencies* of an object are the frequencies at which standing waves may be created in the object.

For a string of mass $m$ and length $L$ fixed at both ends with tension $F_T$, the natural frequencies are given by

$$f_n = n\cdot\frac{v_{string}}{2L}$$

$$\text{where } v_{string} = \sqrt{\frac{F_T}{m/L}}$$

For a tube of length $L$ open at both ends,

$$f_n = n\cdot\frac{v_{sound}}{2L}$$

For a tube of length $L$ open at only one end,

$$f_n = (2n-1)\cdot\frac{v_{sound}}{4L}$$

$f_1$ is the *1st harmonic* or the *fundamental freq*.
$f_2$ is the *2st harmonic* or the *1st overtone*.
$f_3$ is the *3rd harmonic* or the *2nd overtone*.

### Resonance

When an object vibrates near by or in contact with a second object, and the frequency of vibration is near one of the natural frequencies of the second object, the second object will start to vibrate at its natural frequency. This is called *resonance*.

Waves (including sound intensity or light intensity) are amplified via resonance.

## SOUND

*Pitch* is related to the frequency of the sound wave. *Volume* is related to the amplitude (half of the pressure difference).

### Speed of Sound

In a gas $v_{sound} = \sqrt{\gamma\, k\, T/m}$ where $\gamma = \frac{c_P}{c_V}$

$k$ is Boltzmann's constant = $1.380658\times10^{-23}$ J/K

In a liquid $v_{sound} = \sqrt{B_{adiabatic}/\rho}$ where B is the adiabatic bulk modulus.

In a solid $v_{sound} = \sqrt{Y/\rho}$ where Y is Young's modulus.

### Sound Intensity

$$I = \frac{Power}{Area}$$

$$\beta = 10\,\mathrm{dB}\log\left(\frac{I}{I_0}\right)$$

Note that an intensity level of 0 decibels is not 0 $\mathrm{W/m^2}$, it is the threshold of human hearing which is $1.0\times10^{-12}$ $\mathrm{W/m^2}$.

If the intensity level increases by 10 dB, the new sound seems approximately twice as loud.

### The Doppler Effect

The *Doppler effect* is the change in frequency or pitch of a sound detected by an observer due to the relative motion of the source and the observer relative to the medium of sound propogation.

Source moving toward stationary observer:

$$f_o = f_s\left(\frac{1}{1-\frac{v_s}{v}}\right)$$

Source moving away from stationary observer:

$$f_o = f_s\left(\frac{1}{1+\frac{v_s}{v}}\right)$$

Observer moving toward stationary source:

$$f_o = f_s\left(1+\frac{v_o}{v}\right)$$

Observer moving away from stationary source:

$$f_o = f_s\left(1-\frac{v_o}{v}\right)$$

General:

$$f_o = f_s\left(\frac{1\pm\frac{v_o}{v}}{1\mp\frac{v_s}{v}}\right)$$

# AP Physics B Review Sheet

## LIGHT

*Electromagnetic radiation*, including *visible light*, is produced by vibrating electric charges. This creates an oscillating electric field perpendicular to the direction of propagation. The current formed by the moving electric charges also creates an oscillating magnetic field perpendicular to both the direction of propagation and the electric field.

Scientists now agree that light has a dual nature, part particle and part wave. According to this theory, light also consists of massless bundles of concentrated electromagnetic energy called *photons*.

Whether light appears to be a particle or a wave depends on what is being measured and/or how the experiment is designed.

### The Electromagnetic Spectrum

Radio Waves - low frequency, long wavelength
Microwaves
Infrared
Visible Light (ROYGBIV)
Ultraviolet
X-rays
Gamma Rays – high frequency, short $\lambda$

### The Speed of Light

$$c=\frac{1}{\sqrt{\epsilon_0\mu_0}}=299{,}792{,}458\,m/s$$

### Polarization

In *polarized light*, all of the oscillations of the electric field are in the same plane. This plane is called the *direction of polarization*.

In *unpolarized light*, the direction of polarization is not fixed, but fluctuates randomly in time. The direction of oscillation of the electric field is different for different photons.

*Polarizing materials* allow only the component of the wave's electric field along one direction to pass through. This preferred transmission direction for the electric field is called the *transmission axis*.

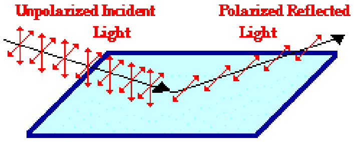
When unpolarized light is incident on a piece of polarizing material, the transmitted polarized light has an intensity that is one-half that of the incident light.

When two pieces of polarizing material are used one after the other, the first is called the *polarizer* and the second is called the *analyzer.*

If the average intensity of polarized light hitting the analyzer is $S_0$, then the average intensity of light leaving the analyzer is

$$S=S_0\cos^2\theta$$

where $\theta$ is the angle between the transmission axes of the polarizer and analyzer.



**Reflection of light off of non-metallic surfaces results in some degree of polarization parallel to the surface.**

The angle of incidence at which the reflected light is completely polarized is parallel to the surface is called *Brewster's angle*.

$$\tan\theta_B=\frac{n_2}{n_1}$$

At this angle, the reflected and refracted rays are perpendicular to each other.

### Shadows

A thin beam of light is often called a *ray*. Any beam of light, no matter how wide, can be thought of as made of a bundle of light rays.

A *shadow* forms where light rays cannot reach. Sharp shadows are produced by a small light source nearby or by a large light source further away.Fuzzy-edged shadows are formed by larger light sources close to an object, because light rays from one part of the light source may be blocked while others reach that part of the shadow.

The darkest part of the shadow, where no light reaches, is called the *umbra*. The lighter area of partial shadow is called the *penumbra*.

### Opacity and Transparency

When an electromagnetic wave hits an atom, the electrons in the atom are forced into vibration. The natural frequency of an electron depends on how strongly it is held by a nearby nucleus.

When an electromagnetic wave encounters an electron the electron may absorb the light as it jumps up energy levels.

If the frequency of the light was the same as the electron's natural frequency, the electron holds on to this energy for a longer time (about 100 millionths of a second). During this time the atom collides with its neighbors many times and the gives up this energy as heat.

If the frequency of the light is not similar to the electron's natural frequency, it emits the energy quickly as light. The frequency of the light may change depending on what energy level the electron drops into.

If the light is emitted in the same direction it was originally traveling in, the material is *transparent.* If the light is emitted randomly in a forward direction, but not exactly the same direction, the material is *translucent*. If the light is emitted backwards, the material is *opaque.*

## COLOR

*Color* is provoked by the frequencies of visible light emitted or reflected by things, but it is also in the eye of the beholder as whether or not these frequencies of light are actually perceived as colors depend on the eye-brain system. For instance, many organisms, including people with red-green color blindness, will see no red in a rose.

*White light* is the combination of all frequencies of visible light. *Black* is the absence of light.

### Color by Reflection

When light hits an object light of some frequencies is absorbed by the cells in the object and some light is reflected. The reflected frequencies create the color of the object. Most objects do not have pure single-frequency colors, but are composed of a spread of frequencies.

Note that only colors present is the original light source could be reflected this way, which is why objects look different colors under different light sources.

### Color by Transmission

The color of a transparent object depends on the frequencies of the light it transmits. The material in the object that selectively absorbs colored light is known as a *pigment*, and the frequencies absorbed by the pigment are not transmitted.

From an atomic point of view, electrons in the pigment selectively absorb light of certain frequencies, while other frequencies are transmitted through the glass. The energy in the absorbed light increases the kinetic energy of the atoms, and the object is warmed.

### Additive Color (Lights)

All frequencies in the mix are seen.

The primary colors are red, green, and blue.

When mixing equal amounts of light,
red + green = yellow
red + blue = magenta
green + blue = cyan
red + green + blue = white

### Subtractive Color (Pigments)

Only those frequencies not absorbed by any of the pigments are seen.

The primary colors are magenta, yellow, and cyan.

When mixing equal amounts of pigment,
magenta + yellow = red
magenta + cyan = blue
yellow + cyan = green
magenta + yellow + cyan = grey/black

# AP Physics B Review Sheet

## FLUID MECHANICS

### Static Fluids

*Fluids* are materials that can flow, and include both gases and liquids (and sometimes amorphous solids).

The *mass density* is the mass per unit volume. It is denoted by the Greek letter rho, $\rho$.

The *specific gravity* of a substance is the ratio of its density compared to the density of a common reference material. Specific gravity has no units.

Usually the common reference material is chosen to be water at 4ºC with a density of $1.000 \times 10^3$ kg/m$^3$.

In a static fluid, pressure is exerted perpendicularly to the surface of any object in contact with the fluid.

Pressure depends only on depth and density of the fluid: $P_{bottom} = P_{top} + \rho g h$

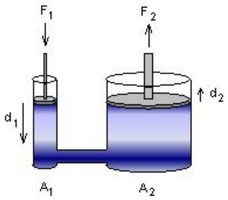
Average atmospheric pressure at sea level is $1.013 \times 10^5$ Pa.

The actual pressure $P$ is known as the *absolute pressure*. The difference $P - P_{atm} = \rho g h$ is known as the *gauge pressure*.

### Pascal's Principle

Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and the enclosing walls.

### Hydraulics



$$\frac{F_2}{A_2} = \frac{F_1}{A_1} + \rho g h$$

### Buoyant Force

The upward force provided to an object wholly or partially immersed in a fluid is called the *buoyant force*. The buoyant force exists because fluid pressure is larger at greater depths.

$$F_B = \rho V g$$

Note that $\rho$ is the density of the liquid, not the density of the object, and $V$ is the volume of displaced liquid (which will equal the volume of the object if it is totally submerged, or may be a fraction of the volume of the object if it is only partially submerged).

### Archimedes Principle

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces.

### Flowing Fluids

In *steady flow* or *laminar flow* the velocity of the fluid particles at any point is constant as time passes. Note that the velocity at different points may be different from one another, but at each point the velocity is constant.

*Unsteady flow* exists whenever the velocity at a point in the fluid changes as time passes.

*Turbulent flow* is an extreme kind of unsteady flow and occurs when there are sharp obstacles or bends in the path of a fast moving fluid. In turbulent flow the velocity at any particular point changes erratically from moment to moment, both in magnitude and direction.

A *viscous fluid* does not flow readily. The viscosity hinders neighboring layers of fluid from sliding freely past one another. The flow of a viscous fluid is an energy-dissipating process. A *nonviscous fluid* flows in an unhindered manner with no dissipation of energy. No real fluid has zero viscosity at normal temperatures, but some fluids have negligibly small viscosities.

An incompressible, nonviscous fluid is called an *ideal fluid*.

### Flow Rates

The mass of fluid per second that flows through a pipe is called the *mass flow rate*.

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t} = \rho A v$$

The quantity $Q = A \cdot v$ is the volume of fluid that passes through the pipe each second and is called the *volume flow rate*. This remains constant as a pipe constricts or expands.

### Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

### Magnus Effect

When an object spins, air close to its surface is dragged around with it by surface irregularities. The air on the side rotating into its direction of motion is slowed down while the air on the side rotating away from the direction of motion is sped up, resulting in pressure differences that create a net force called the *Magnus effect*.

### Torricelli's Law

Suppose you have a large tank, where the surface is at atmospheric pressure, and there is a small hole or open pipe near the bottom of the tank. The speed at which the water exits the tank is called the *efflux speed*.

$$v_{efflux}^2 = v_{surface}^2 + 2gh$$

If the tank is large, then the liquid level changes very slowly and $v_{efflux} \approx \sqrt{2gh}$ Note that this is the speed the water would have had it freely fallen that height difference.

## THERMODYNAMICS

### Temperature

*Temperature* is a description of how hot or cold something is. It can be measured by observing a change in some *thermometric property* of an object.

$$K = {}^\circ C + 273.15$$

$$^\circ F = \frac{9}{5}({}^\circ C) + 32$$

### Absolute Zero

The phrase *absolute zero* means that temperatures lower than -273.15 °C cannot be reached by continually cooling a gas or any other substance. If lower temperatures could be reached, then further extrapolation of the straight line experimentally found on a P-T graph created with a constant volume thermometer would suggest that negative absolute gas pressures could be reached, which is impossible as it has no meaning.

### Thermal Expansion

$\alpha$ is the *coefficient of linear expansion*.

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

$$\text{Stress} = \frac{F}{A} = Y \frac{\Delta L}{L_0}$$

If a heated plate has a hole in it, the hole increases in size in each dimension.

$$\frac{\Delta A}{A_0} \approx 2\alpha \Delta T$$

$\beta$, is called the *coefficient of volume expansion*

$$\frac{\Delta V}{V_0} = \beta \Delta T \approx 3\alpha \Delta T$$

### Heat

*Internal energy* is the sum of the molecular kinetic energy (due to the random motion of molecules), the molecular potential energy (due to forces that act between the atoms of a molecule and forces that act between molecules), and other kinds of molecular energy.

*Heat, Q,* is energy that flows from a higher-temperature object to a lower-temperature object because of the difference in temperatures.

$$Q = m \cdot c \cdot \Delta T$$

$$Q = m \cdot L$$

The proportionality constant, *c*, is the *specific heat capacity* of the material. The *latent heat, L,* is the heat per kilogram associated with a phase change.

Heat transfer will continue until a common temperature, *thermal equilibrium*, is reached.

### Mechanical Equivalent of Heat

1 cal = 4.186 J

# AP Physics B Review Sheet

### Convection

When part of a fluid is warmed, the volume of the fluid expands and its density decreases. According to Archimedes' Principle, the surrounding cooler and denser fluid exerts a buoyant force on the warmer fluid and pushes it upward. As the warmer fluid rises, the surrounding cooler fluid replaces it. This cooler fluid, in turn, is warmed and pushed upward. This creates a continuous fluid flow, called a *convection current*, which carries along heat. The transfer of heat is done by *convection*.

### Conduction

*Conduction* is the process whereby heat is transferred directly through a material, but any bulk motion of the material plays no role in the transfer.

$$Q=\frac{k\cdot A\cdot\Delta T\cdot t}{L}$$

The proportionality constant, *k*, is called the *thermal conductivity* of the material.

Conduction happens best in metals because the free electrons in the metallic bonds transfer heat rapidly through the substance.

### Radiation

The process of transferring energy via electromagnetic waves is called *radiation;* it does not require a material medium.

$$Q=e\cdot\sigma\cdot T^4\cdot A\cdot t$$

The proportionality constant, *σ*, is called the *Stefan-Boltzmann constant*.

$$\sigma = 5.67\times 10-8\,\frac{J}{s\cdot m^2\cdot K^4}$$

The factor *e* is the *emissivity*, which is a number between 0 and 1 that represents the ratio of the energy actually emitted by an object and what it would emit if it were a perfect blackbody.

All bodies continuously radiate energy in the form of electromagnetic waves, though it may be in a form other than visible light.

When a body has the same temperature as its surroundings, the amount of radiant energy being emitted must balance the amount of radiant energy being absorbed.

### Ideal Gas Law

An *ideal gas* is an idealized model for real gases that have sufficiently low densities. This condition means that the molecules of the gas are so far apart that they do not interact other than via collisions that are effectively elastic.

$$P\cdot V=n\cdot R\cdot T$$

$$P\cdot V=N\cdot k\cdot T$$

The proportionality constant is the *universal gas constant*, *R*, which has been experimentally determined to be 8.31 J/(mol K). Related to this is *Boltzmann's constant*, *k* = *R* / Å which is $1.38\times10^{-23}$ J/K.

### Pressure of a Gas (Molecular Scale)

$$P=\left(\frac{N}{3}\right)\left(\frac{m\,v_{rms}^2}{L^3}\right)=\left(\frac{N}{3}\right)\left(\frac{m\,v_{rms}^2}{V}\right)$$

### Kinetic Energy of a Gas (Molecular Scale)

$$\overline{KE}=\frac{1}{2}m\,v_{rms}^2=\frac{3}{2}k\,T$$

### Internal Energy of a Gas

$$U=N\left(\frac{1}{2}m\,v_{rms}^2\right)=N\left(\frac{3}{2}k\,T\right)=\frac{3}{2}n\,R\,T$$

### Thermodynamics

The *system* is the collection of objects upon which attention is being focused. Everything else in the environment is called the *surroundings*.

The physical condition of the system is called the *state* of the system. It includes pressure, volume, temperature, and mass of the system.

The system and its surroundings must be separated by walls of some kind. Walls that permit the transfer of heat are called *diathermal walls*. Perfectly insulating walls that do not permit the flow of heat from the system to the surroundings are called *adiabatic walls*.

### The Zeroth Law of Thermodynamics

Two systems individually in thermal equilibrium with a third system are in thermal equilibrium with each other. Objects in thermal equilibrium will have the same temperature.

### The First Law of Thermodynamics

$$\Delta U=U_f-U_i=Q-W$$

*Q* is positive when the system gains heat
*Q* is negative when the system loses heat
*W* is positive when work is done by the system
*W* is negative when work is done on the system

Because internal energy depends only on temperature, *ΔU* is determined once the initial and final temperatures are known.

Internal energy depends only on the state of a system, not on the method by which the system arrives at a given state.

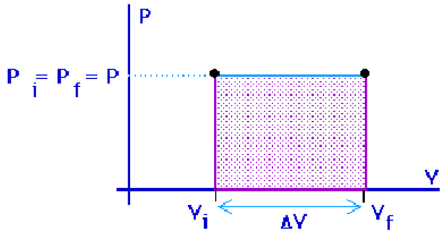
### Pressure-Volume Graphs

The area under the curve on a pressure-volume graph is the work for any kind of process.

### Isobaric Process

An *isobaric process* is one that occurs at constant pressure.

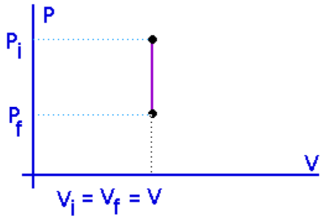
$$W=P\cdot\Delta V$$



### Isochoric Process

An *isochoric process* is one that occurs at constant volume.

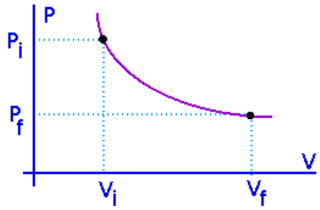
$$W=0$$



### Isothermal Process

An *isothermal process* is one that occurs at constant temperature.

$$W=nRT\ln\left(\frac{V_f}{V_i}\right)=Q$$



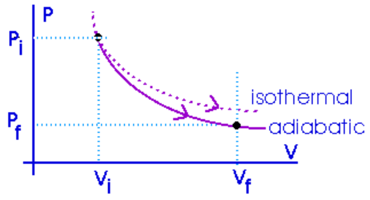
$$W_{isotherm}=n\,R\,T\ln\left[\frac{V_f}{V_i}\right]$$

### Adiabatic Process

An *adiabatic process* is one that occurs without any heat flow between the system and the surroundings.

$$Q=0$$

$$W=\frac{3}{2}nR\left(T_i-T_f\right)$$



$$P_iV_i^{\gamma}=P_fV_f^{\gamma}$$

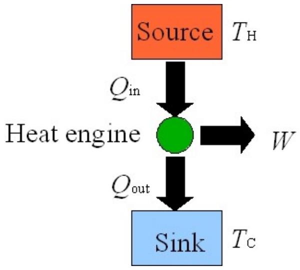
$$\gamma=\frac{c_P}{c_V}$$

monotomic ideal gas: $\gamma=5/3$
diatomic ideal gas: $\gamma=7/5$

# AP Physics B Review Sheet

**The Second Law of Thermodynamics**
Heat flows spontaneously from a substance at a higher temperature to a substance at a lower temperature, and does not flow spontaneously in the reverse direction.

**Heat Engines**
A *heat engine* is any device that uses heat to perform work.



The *efficiency*, *e*, of a heat engine is defined as the ratio of the work done by the engine to the input heat.

$$e = \frac{W}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

A *reversible process* is one in which both the system and its environment can be returned to exactly the states they were in before the process occurred.

A process that involves an energy-dissipating mechanism or a spontaneous process cannot be reversible because the energy wasted would alter the system, the environment, or both.

**Carnot's Principle**
No irreversible engine operating between two reservoirs at constant temperatures can have a greater efficiency than a reversible engine operating between the same temperatures. Furthermore, all reversible engines operating between the same temperatures have the same efficiency.

An important feature of a reversible engine, called a *Carnot engine*, is that all of the input heat originates in a hot reservoir at a uniform temperture of $T_H$ and all the waste heat goes into a cold reservoir at a uniform temperature of $T_C$ .

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \qquad e = 1 - \frac{T_C}{T_H}$$

**Refrigeration**
If work is used, heat can be made to flow from cold to hot, against its natural tendency. The process of removing heat from the cold reservoir and adding it to the hot reservoir is called a *refrigeration process*.

$$\text{Coefficient of performance} = \frac{Q_C}{W}$$

If the process occurs reversibly, we have an ideal device called a Carnot refrigerator or Carnot air conditioner.

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \qquad \begin{matrix}\text{Coefficient}\\ \text{of performance}\end{matrix} = \frac{T_C}{T_H - T_C}$$

**Heat Pumps**
The term *heat pump* is reserved for a home heating device which used work $W$ to make heat $Q_C$ from the wintry outdoors flow up the temperature hill into a warm house.

$$\text{Coefficient of performance} = \frac{Q_H}{W}$$

If the process occurs reversibly, we have an ideal device called a Carnot heat pump.

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \qquad \begin{matrix}\text{Coefficient}\\ \text{of performance}\end{matrix} = \frac{T_H}{T_H - T_C}$$

**Entropy**
Irreversible processes lose some ability to perform work. This partial loss can be expressed in terms of *entropy.*

The quantity $Q / T$ is called the *change in entropy* and applies to any process in which heat $Q$ enters or leaves a system reversibly at a constant temperature $T$.

$$\Delta S = \left(\frac{Q}{T}\right)_R$$

Entropy, like internal energy, is a function of the state or condition of they system. Only the state of a system determines the entropy S that a system has. Therefore, the change in entropy $\Delta S$ is equal to the entropy of the final state minus the entropy of the initial state.

**The Second Law and Entropy**
The Second Law of Thermodynamics states that if the physical process is irreversible, the combined entropy of the system and the environment must increase. The final entropy must be greater than the initial entropy for an irreversible process:

$S_f > S_i$ (irreversible process)

When a reversible process occurs, the combined entropy of the system and the environment does not change.

$S_f = S_i$ (reversible process)

When an irreversible process occurs, and the entropy of the universe increases, the energy for doing work decreases.

$$W_{unavailable} = T_C \Delta S_{universe}$$

We associate an increase in entropy with an increase in disorder, and a decrease in entropy with a decrease in disorder (or a greater degree of order).

**The Third Law of Thermodynamics**
It is not possible to lower the temperature of any system to absolute zero in a finite number of steps.

## ELECTROSTATICS

**Conservation of Electric Charge**
During any process, the net electric charge of an isolated system remains constant.

Usually electrons are transferred rather than protons, because they take less energy to move as they are on the outside of the atom.

Whenever two different materials rub against each other it is likely that one will leave with more electrons than it started with...the other will leave with less. This is called *triboelectricity*.

> When a rubber rod is rubbed with animal fur, some of the electrons in the fur transfer to the rubber rod.
>
> If a glass rod is rubbed with silk cloth, some of the electrons in the glass transfer to the silk.

Items that allow the easy flow of electrons are called *electrical conductors*. Most metals are conductors because of the nature of metallic bonds.

Items that inhibit the flow of electrons are called *electrical insulators*. Most nonmetals are insulators because of the nature of covalent bonds and ionic bonds in solids.

Rubbing two objects together to make an electrical imbalance is called *charging by friction*.

Transferring electrons from one material to another by simply touching is called *charging by contact*.

If we bring a charged object near a conducting surface, even without physical contact, electrons will move in the conducting surface. This can be used to charge the object by *induction*, if the object is grounded.

*Charge polarization* occurs when a charged rod is brought near an insulator. There is a rearrangement of the position of charges within the atoms and molecules themselves.

**Coulomb's Law**
The electrical force between any two point-charges with charges $q_1$ and $q_2$, separated by a distance $r$ obeys an inverse-square law:

$$\vec{F}_e = k\frac{q_1 q_2}{r^2}$$

The constant $k$ is often expressed in terms of a more fundamental constant called the *permittivity of free space*,

$$\epsilon_0 = 8.854187817 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.987551788 \times 10^9 \frac{\mathrm{N \cdot m}^2}{\mathrm{C}^2}$$

# AP Physics B Review Sheet

### Electric Fields

The electric field at the location of a point charge $q_0$ is

$$\vec{E}=\frac{\vec{F}_e}{q_0}$$

The electric field due to a point charge $q$ is

$$\vec{E}=\frac{kq}{r^2}=\frac{q}{4\pi\epsilon_0 r^2}$$

If $q$ is positive, then $E$ is directed away from $q$
If $q$ is negative, then $E$ is directed toward $q$

### Electric Field Inside Conductors

At equilibrium under electrostatic conditions, any excess charge will reside on the surface of a conductor, and the electric field is zero at any point within a conducting material.

### Gauss' Law for a Point Charge

The product of the magnitude of the electric field at any point on the Gaussian surface and the area of the surface is called the electric flux,

$$\Phi_E=EA=\frac{q}{\epsilon_0}$$

### Gauss' Law

Suppose we have a charge distribution whose net charge is $Q$. The charge distribution is surrounded by a *Gaussian surface* with any arbitrary closed shape. The direction of the electric field need not be perpendicular to the surface, and the magnitude of the electric field may vary on the surface.

$$\Phi_E=\Sigma E\cos(\phi)\Delta A=\frac{Q}{\epsilon_0}$$

Or, using calculus

$$\Phi_E=\oint\vec{E}\cdot d\vec{A}=\frac{Q}{\epsilon_0}$$

### Electric Potential

The *electric potential, V,* at a given point is the electric potential energy EPE of a small test charge $q_0$ at that point divided by the charge.

$$V=\frac{EPE}{q_0}$$

The *electric potential difference* between two points is related to the work per unit charge involved in moving a charge between those two points.

$$\Delta V=\frac{\Delta EPE}{q_0}=\frac{-W_{AB}}{q_0}$$

### Potential Difference from a Point Charge

$$V=\frac{kq}{r}$$

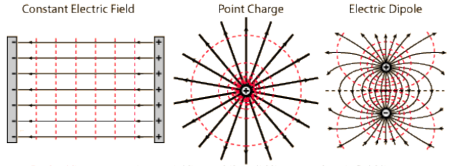
relative to a potential of 0 V at infinity.

When two or more charges are present, the potential due to all the charges is obtained by adding together the individual potentials.

### Equipotential Surfaces

An *equipotential surface* is a surface on which the electric potential is the same everywhere.

The net electric force does no work as a charge moves on an equipotential surface.



Dashed lines are equipotential lines while solid lines are electric field lines.
Click on one of the diagrams for further detail.

### Parallel Plate Capacitors

A *parallel plate capacitor* consists of two metal plates, each with area $A$. A charge $+q$ is spread uniformly over one plate, while a charge $-q$ is spread uniformly over the other plate.

The electric field between the plates is

$$\vec{E}=\frac{kq}{r^2}=\frac{q}{\epsilon_0 A}=\frac{\sigma}{\epsilon_0}$$

where $\sigma$ is the charge per unit area (also called the *charge density*). Except near the edges, the field has the same strength at all places between the plates, and the field does not depend on the distance from the edges.

The potential difference between the capacitor places is

$$\Delta V=-E\Delta s$$

$\Delta s$ is the displacement along a line perpendicular to the plates

$$E=-\frac{\Delta V}{\Delta s}$$

$\Delta V/\Delta s$ is called the *potential gradient.*

The amount of charge on the plates is proportional to the potential difference between the plates: $q=C\cdot V$ The constant $C$ is the *capacitance* of the capacitor.

It is common practice to fill the region between the conductors or plates with an electrically insulating material called a *dielectric*. A dielectric reduces the electric field strength between the plates, allowing for more charge to be stored on them at the same potential.

$$\kappa=\frac{E_0}{E}$$

$E_0$ is the field magnitude without the dielectric
$E$ is the field magnitude with the dielectric

A dielectric will increase the capacitance of a capacitor: $C=\kappa\cdot C_0$
$C_0$ is the capacitance without the dielectric
$C$ is the capacitance with the dielectric

The energy stored is $E=\frac{1}{2}CV^2$

### Energy Density of an Electric Field

$$\text{Energy Density}=\frac{EPE}{V}=\frac{1}{2}\kappa\epsilon_0 E^2$$

## ELECTRIC CURRENT

### Motion of Charged Particles in a Potential

Positive charges will accelerate from a region of high potential to a region or lower potential.

Negative charges will accelerate from a region of low potential to a region of higher potential.

### Electromotive Force

Because of the positive and negative charges on the battery terminals, and electric potential difference exists between them. The maximum potential difference is called the *electromotive force*, *emf*, of the battery, for which the symbol $\mathscr{E}$ is used.

Note that generally the potential difference between the terminals of a battery is a bit less than the maximum value indicated by the emf.

### Conventional Current vs. Electron Flow

Positive charges are repelled from the positive terminal and travel through the wire toward the negative terminal. This is *conventional current*.

We are now aware that it is electrons that move, not positive charges, but we continue to use conventional current.

### Current

In a circuit the battery creates an electric field within and parallel to the wire, directed from the positive to the negative terminal. This field exerts a force on the free electrons in the wire, and they respond by moving from the negative terminal to the positive terminal. This flow of charge is known as an *electric current.*

$$I=\frac{\Delta q}{\Delta t}$$

If the charges move around the circuit in the same direction at all times, the current is said to be *direct current.*

If the charges move first one way, and then the opposite way, changing direction from moment to moment, the current is said to be *alternating current.*

### Resistance

The *resistance*, $R$, is the ratio of the voltage, $V$, applied across a piece of material to the current, $I$, through the material.

$$R=\frac{V}{I}$$

**Ohm's Law** (does not apply universally)

$$V=I\cdot R$$

### Electrical Power

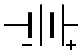
$$P=I\cdot V=I^2\cdot R=\frac{V^2}{R}$$

# AP Physics B Review Sheet

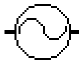
## ELECTRIC CIRCUITS

### Symbols

Battery (DC)

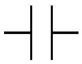


Generator (AC)



Resistor

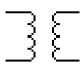
Capacitor



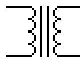
Inductor (Air Core)

Inductor (Iron Core)

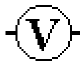
Transformer (Air Core)



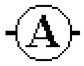
Transformer (Iron Core)



Voltmeter



Ammeter



### Resistors

To the extent that a wire or electrical device offers resistance to the flow of charges, it is called a *resistor.*

Resistors play an important role in electric circuits, where they are used to limit the amount of current and establish proper voltage levels.

The electrical resistance of a piece of material of length *L* and cross-sectional area *A*, is

$$R=\rho\left(\frac{L}{A}\right)$$

The proportionality constant, *ρ*, is the *resistivity* of the material.

The resistivity of a material depends on temperature. In metals the resistivity increases with increasing temperature, while in semiconductors, the resistivity increases with decreasing temperature.

$$\rho=\rho_0\left[1+\alpha\left(T-T_0\right)\right]$$

The term α has the unit of reciprocal temperature and is the *temperature coefficient of resistivity.*

### Internal Resistance

In a battery, the *internal resistance* comes from the chemicals within the battery. In a generator, the internal resistance comes from the resistance of the wires and other components within the generator.

The internal resistance causes the voltage between the terminals to drop below the maximum value specified by the battery's emf. This actual voltage is known as the *terminal voltage*.

### Series Wiring

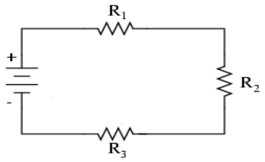
*Series* wiring means that the devices have been connected so that all the current flows through each device.

The current through each device in a series circuit is the same.

The voltage will drop through each device, to be built up again by the battery or other emf source.

Series resistors: $R_S=R_1+R_2+R_3+\cdots$

Series capacitors: $\frac{1}{C_S}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots$



### Parallel Wiring

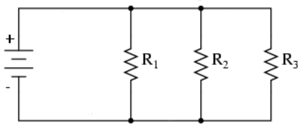
*Parallel* wiring means that the devices have been connected so that the same voltage is applied across each device.

The current into a parallel circuit is split between each device.

The voltage applied to each device in the parallel circuit is the same.

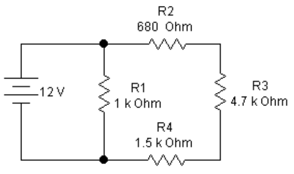
Parallel resistors: $\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots$

Parallel capacitors: $C_P=C_1+C_2+C_3+\cdots$



### Circuits Wired Partially in Series and Partially in Parallel

1. Draw a schematic diagram.
2. Start with the most embedded portion of the circuit and calculate a single equivalent resistance for those resistors. Draw a new schematic.
3. Repeat until you can reduce the circuit to a single resistor. Find the total circuit current and then go back through the circuits to find the currents and voltages across individual resistors.



### Kirchhoff's Rules

There are many circuits in which no two resistors are in series or in parallel. In that case, we need to use Kirchhoff's Rules.

*Kirchhoff's Current Law (Junction Rule)*
The sum of the magnitudes of the currents directed into a junction equals the sum of the magnitudes of the currents directed out of the junction.

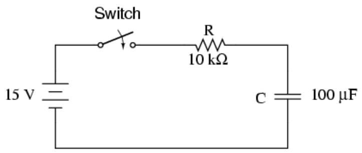
*Kirchhoff's Voltage Law (Loop Rule)*
Around any closed circuit loop, the sum of the potential drops equals the sum of the potential gains.

*Using Kirchhoff's Rules*

1. Assume all voltage sources and resistances are given. (If not label them V1, V2, ..., R1, R2, etc)
2. Label each branch with a branch current. (I1, I2, I3, etc)
3. Find Kirchoffs first law equations for each node.
4. Find Kirchoffs second law equations for each of the independent loops of the circuit.
5. Solve the simultaneous equations as required to find the unknown currents.

### RC Circuits

Many electric circuits contain both resistors and capacitors.



When the switch is closed, the battery begins to deposit charge on the capacitor plates. The resistor slows down this process.

Assuming that the capacitor is uncharged at time $t = 0$ s when the switch is closed, and it is connected to a potential difference $V_0$, it can be shown that the magnitude *q* of the charge on the plates at time *t* is

$$q=q_0\left(1-e^{-t/RC}\right),\ \text{where}\ q_0=CV_0$$

The voltage across the capacitor at time *t* is

$$V=V_0\left(1-e^{-t/RC}\right)$$

When a circuit containing a capacitor is disconnected from the voltage source, the capacitor will send charge through the circuit and power it, until the capacitor is fully discharged.

$$q=q_0e^{-t/RC}$$
$$V=V_0e^{-t/RC}$$

The term *RC* in the exponent is called the *time constant, τ,* of the circuit.

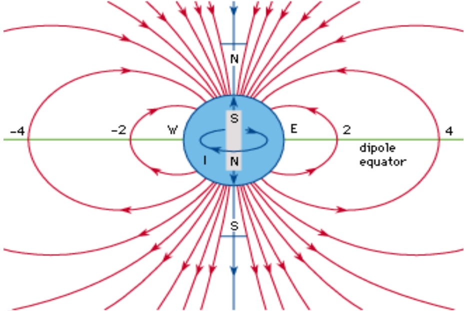
# AP Physics B Review Sheet

## MAGNETISM

Like poles on different magnets repel each other; unlike poles attract.

### Magnetic Field

Surrounding a magnet is a three-dimensional magnetic field. The direction of the magnetic field at any point in space is the direction indicated by the north pole of a small compass needle placed at that point.



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The magnitude $B$ of the magnetic field at a point in space is defined as

$$B=\frac{F}{q_0(v\sin\theta)}$$

where $F$ is the magnitude of the magnetic force on a positive test charge $q_0$ and $v$ is the velocity of the charge and makes an angle $\theta$ with the direction of the magnetic field.

The strength of the magnetic field near the Earth's surface is about $1\times10^{-4}$ T, also known as a gauss, G.

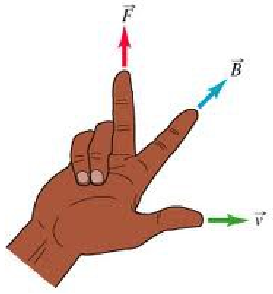
### Magnetic Force

When an electric charge is placed in a magnetic field, it experiences a force provided certain conditions are met:

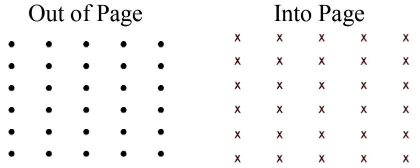
1. The charge must be moving relative to the magnetic field
2. The velocity of the moving charge must have a component that is perpendicular to the direction of the magnetic field

$$\vec{F}_B=q\vec{v}\times\vec{B}$$

$$F_B=qvB\sin\theta$$



### Drawing Magnetic Fields



### Lorentz Force (Electric and Magnetic)

$$\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}$$

$$F=qE+qvB\sin\theta$$

When an electric force is applied to a positively charged particle, the path of the particle bends in the direction of the force. Because there is a component of the particle's displacement in the direction of the electric force, the force does work on the particle.

When a magnetic force is applied to a positively or negatively charged particle, it always acts in a direction that is perpendicular to the motion of the charge. Consequently, the magnetic force cannot do work on the particle and change its kinetic energy, although it does alter the direction of the motion by providing a centripetal force.

$$qvB\sin(90°)=\frac{mv^2}{r}$$

$$r=\frac{mv}{qB}$$

### Magnetic Force on a Long, Straight Wire

$$\vec{F}=\vec{I}L\times\vec{B}$$

$$F=ILB\sin\theta$$

### Torque on a Current Carrying Loop

If the wire is wrapped to form a coil containing $N$ loops, each of area $A$, the net torque is

$$\Sigma\tau=NIAB\sin\theta$$

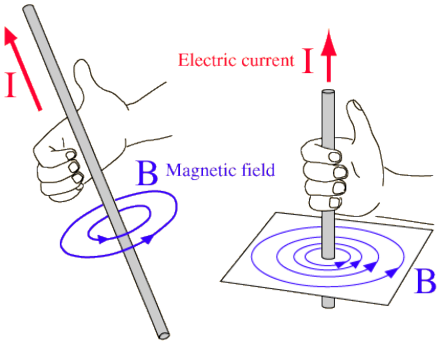
The quantity $NIA$ is known as the *magnetic moment* of the coil, and its units are A·m$^2$.

### Magnetic Field Produced by a Wire

$$B=\frac{\mu_0 I}{2\pi r}$$

$$\mu_0=4\pi\times10^{-7}\,\mathrm{T\cdot m/A}$$

The constant $\mu_0$ is known as the *permeability of free space*.



### Loop of Wire

If a current-carrying wire is bent into a circular loop with $N$ turns, the magnetic field lines around the loop are concentrated in the center and loop radially around the loop.

At the center $$B=N\frac{\mu_0 I}{2R}$$

Along axis $$B=N\frac{\mu_0 2\pi R^2 I}{4\pi(r^2+R^2)^{3/2}}$$

### Solenoids

A *solenoid* is a long coil of wire in the shape of a helix.

If the wire is wound so that the turns are packed close to each other and the solenoid is long compared to the diameter, the magnetic field lines inside are are nearly constant in magnitude and directed parallel to the axis.

$$B=\mu_0 nI$$

The magnitude of the magnetic field outside the solenoid is not constant and is much weaker than the interior field. In fact, the magnetic field outside is nearly zero if the length of the solenoid is much greater than its diameter.

### Ampère's Law

The general law known as Ampère's Law gives the magnetic field at any point around a wire of any geometrical shape.

Consider any arbitrary closed path around a current, and imagine it as being made up of short segments of length $\Delta l$. We take the product of the length of each segment times the component of the magnetic field parallel to that segment. If we sum all these terms, the result is the product of $\mu_0$ and the net enclosed current $I_{\mathrm{enc}}$.

$$\Sigma B_{//}\Delta l=\mu_0 I_{enc}$$

If you let the length $\Delta l$ go to zero, then this becomes $\oint\vec{B}\times d\vec{l}=\mu_0 I_{enc}$

### Atomic Explanation for Magnetism

Electrons orbiting the nucleus behave like atomic sized loops of current. Each electron has a spin that also gives rise to a magnetic field.

In most substances the magnetism produced at the atomic level tend to cancel out, with the result that the substance is nonmagnetic overall.

*Ferromagnetic materials* are materials where cancellation of the atomic magnetic fields does not occur for groups of approximately $10^{16}$ to $10^{19}$ neighboring atoms, because they have electrons spins that are naturally aligned parallel to each other.

This alignment results in a special type of quantum mechanical interaction between spins. The result is a small but highly magnetized region of about 0.01 to 0.1 mm in size, called a *magnetic domain*.

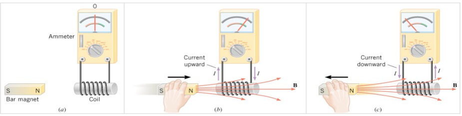
The magnetic domains can be forced to align by placing the object in an external magnetic field. The domains who magnetism is parallel or nearly parallel to the external field grow in size by absorbing unaligned domains, while the magnetic alignment of other domains may rotate and become more oriented in the direction of the external field.

# AP Physics B Review Sheet

## ELECTROMAGNETIC INDUCTION

### Induced EMF and Induced Current

When a magnet moves relative to a coil of wire, an ammeter connected to the coil will read a positive or negative current, depending on the direction of motion of the magnetic field. Since a source of electromotive force, emf, is always needed to produce a current, the coil itself behaves as if it were a source of emf. This emf is known as an *induced emf*. The current caused by the induced emf in the coil is called an *induced current*.



### Motional EMF

When a conducting rod moves through a constant magnetic field, an emf is induced in the rod. The charged particles in the conductor are carried along with the moving conductor, so they experience a force from the magnetic field that causes positive charges to pile up on one end of the conductor and negative charges to pile up on the other end. This separation of charge creates an electric field inside the conductor. The charges that pile up create a voltage or emf across the length of the rod that is constant. This induced emf is also called a *motional emf*.

$$qE = qvB\sin 90°$$
$$q\left(\frac{\mathcal{E}}{L}\right) = qvB$$
$$\mathcal{E} = vBL$$

### Magnetic Flux

The *magnetic flux* $\Phi_B$ for a uniform magnetic field through a loop of area $A$ is defined as

$$\Phi_B = \vec{B}\cdot\vec{A} = B_\perp A = B\,A\cos\phi$$

If the area is not a flat surface, or if the magnetic field is not uniform, then the magnetic flux is defined as

$$\Phi_B = \int \vec{B}\cdot d\vec{A}$$

### Faraday's Law of Electromagnetic Induction

Whenever there is a change in flux (over time) through a loop of wire, an emf is induced in the loop.

$$\mathcal{E} = -\Delta\Phi_B/\Delta t$$

The minus sign reminds us that the induced emf will oppose the change in the magnetic flux.

Faraday's law states that an emf is generated if the magnetic flux changes for any reason. Since $\Phi = \text{BA}\cos\theta$, any change of B, A, or $\theta$ will induce an emf.

If the circuit contains $N$ closely wrapped loops, the emfs induced in each loop add together, so

$$\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$$

### Induced Magnetic Fields

A changing magnetic field produces an induced emf which drives a current around a circuit. This induced current produces an induced magnetic field which opposes the change in the original magnetic field.

### Lenz's Law

A current produced by an induced emf moves in a direction so that its magnetic field opposes the original change in flux.

*Determining the Polarity of the Induced EMF*

1. Determine whether the magnetic flux that penetrates a coil is increasing or decreasing.
2. Find what the direction of the induced magnetic field must be so that it can oppose the change in flux by adding to or subtracting from the original field.
3. Use the RHR-2 to determine the direction of the induced current. The polarity of the induced emf can be assigned because conventional current is directed out of the positive terminal, through the external circuit, and into the negative terminal.

### Mutual Inducatance

When an ac current is passed through a primary coil, it generates a magnetic field. If this changing magnetic field penetrates nearby secondary coil, the secondary coil experiences a changing magnetic flux and an induced emf and induced current appears.

The effect in which a changing current in one circuit induces an emf in another circuit is called *mutual induction*.

The induced emf in the secondary coil is proportional to the magnetic flux in the secondary coil, which is proportional in turn to the change in current in the primary coil. We introduce a proportionality constant, $M$, called the *mutual inductance*, which is usually measured experimentally.

$$N_S\Phi_S = M\,I_P$$
$$\text{E}_S = -N_S\frac{\Delta\Phi_S}{\Delta t} = -M\frac{\Delta I_P}{\Delta t}$$

### Self Inductance

An emf can be induced in a current-carrying coil by a change in the magnetic field that the current itself produces. This is referred to as *self-induction*.

Suppose a coil is attached to an ac generator. The alternating current creates an alternating magnetic field that creates a changing flux through the coil. The change in flux induces an emf in the coil in accord with Faraday's Law.If $\Phi$ is the magnetic flux through one turn of the coil, then $N\,\Phi$ is the net flux through the coil with $N$ turns. Since $\Phi$ is proportional to the magnetic field, and the magnetic field is proportional to the current $I$, we can state $N\Phi = L\,I$ where $L$ is the constant of proportionality and is called the *self inductance* of the coil.

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

### Inductors

Because of their self-inductance, coils are known as *inductors*. Like capacitors, inductors can store energy in a circuit. This stored energy arises because a generator does work to establish a current in the inductor.

$$E = \frac{1}{2}L\,I^2$$

where $I$ is the final current through the indcutor.

For the special case of a long solenoid, the self inductance is $L = \mu_0 n^2 A\,l$ where $n$ is the number of turns per unit length, $A$ is the cross-sectional area, and $l$ is the length of the solenoid. Thus the energy stored is

$$E = \frac{1}{2\mu_0}B^2 A\,l$$

### Energy Density of a Magnetic Field

$$\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2\mu_0}B^2$$

### Transformers

A *transformer* is a device for increasing or decreasing an ac voltage. It consists of a ferromagnetic core on which two coils are wound. The primary coil is on the generator side with $N_p$ turns, and a secondary coil on the appliance side with $N_s$ turns.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

In a *step-up transformer*, the number of secondary coils is larger than the number of primary coils, so the secondary voltage is higher than the primary voltage.

In a *step-down transformer*, the number of secondary coils is smaller than the number of primary coils, so the secondary voltage is lower than the primary voltage.

The ratio $N_s$:$N_p$ is known as the *turns ratio* of the transformer.

# AP Physics B Review Sheet

## QUANTUM MECHANICS

### Black Body Radiation

A *black-body* is an object that completely absorbs all of the electromagnetic radiation falling on it. Thus it also emits perfectly too.

When an object is heated it emits radiation consisting of electromagnetic waves (microwaves, infrared, visible light, etc.) with a wide range of frequencies. This explains why heated objects appear to glow dull red, cherry red, orange, yellow, or white as they get hotter.

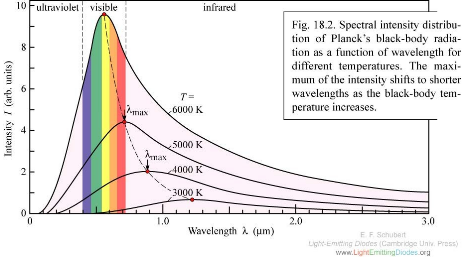


Fig. 18.2. Spectral intensity distribution of Planck's black-body radiation as a function of wavelength for different temperatures. The maximum of the intensity shifts to shorter wavelengths as the black-body temperature increases.

E. F. Schubert
*Light-Emitting Diodes* (Cambridge Univ. Press)
www.LightEmittingDiodes.org

The English physicists Lord Rayleigh and Sir James Jeans derived an equation that agreed well with experiments, but only at the low-frequency, long-wavelength (infrared) end of the spectrum. The classical theory predicted an infinite intensity for the ultraviolet region and beyond. This was dubbed the *ultraviolet catastrophe*.

The German physicist Maxwell Planck assumed that there was some electric oscillator in objects that vibrated at higher and higher frequencies as the object was heated. With this assumption, he found a formula that matched the experimental data, but lacked a physical reality.

Using a trick from calculus, he broke the energies up into small discrete bits proportional to the oscillator frequencies, namely $E = hf$. However, if he allowed the energy chunks to go to zero as the procedure demanded, the equation simplified to the incorrect Rayleigh-Jeans formula. BUT, if he did not require that the energies e or the constant h go to zero, but remained finite, he obtained his own radiation formula which matched experimental evidence exactly!

Planck had stumbled across a theoretical basis for his experimental radiation law, but only if the energy is discontinuous. Even though he had no reason to accept this notion (and he hated its implications), he accepted it provisionally for he had nothing better.

The small, discrete bit of energy is called an *energy quanta*. Planck's constant is

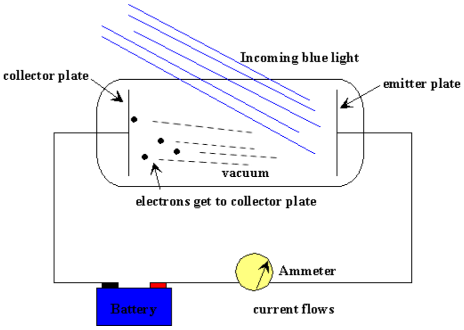
$$h = 6.626\ 069\ 91 \times 10^{-34}\ \text{J·s}$$

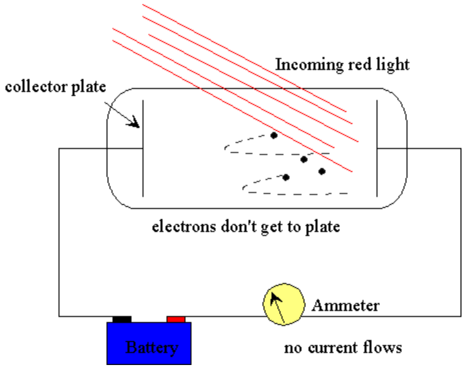
### Photoelectric Effect

The variable voltage source turns the collector plate into a cathode with a surplus of electrons and the emitter into an anode with a lack of electrons, creating a retarding voltage in the vacuum tube that tries to force electrons back toward the emitter plate.

When a light source is turned on, some of the remaining electrons in the anode are ejected. If their kinetic energy is enough to overcome the retarding voltage, they make it to the collector plate, the circuit is completed, and the ammeter measures a current.

The electrons that make the journey and complete the circuit must have had energy greater than $q\,V_0$ where $q$ is the charge of the electron and $V_0$ is the voltage value where the current stops entirely.





There is a well defined minimum voltage, $V_0$ that stopped any electrons getting through; $V_0$ does not depend at all on the intensity of the light!

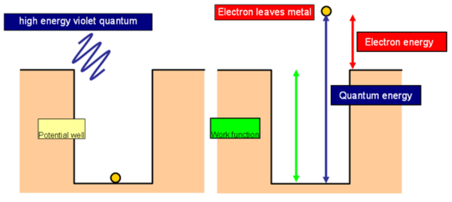
Doubling the light intensity doubles the number of electrons emitted, which doubles the current, but did not affect the energies of the emitted electrons.

He found that the maximum energy of the ejected electrons did depend on the frequency (color); shorter-wavelength higher-frequency light caused electrons to be ejected with more energy.

He also discovered that there is a certain threshold frequency $f_t$ that depends on the type of metal, below which no photoelectrons were ejected, no matter how bright the light beam. Einstein showed that the puzzling features of the photoelectric effect are easily explained once the illuminating radiation is understood to be a collection of particles, or *photons*.

The photons have energy quanta of magnitude $hf$. These energy packets penetrate the surface layer of the metal of the target electrode and hit an electron.

The photon's energy is transformed into the kinetic energy of the electron, and some are ejected. Note that in order to be ejected, each electron must do an amount of work to climb out of the atom to get into free space.



(Diagram: resourcefulphysics.org)

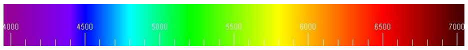
$$KE_{photoelectron} = E_{photon} - W$$
$$q\,V_0 = h \cdot f_{photon} - h \cdot f_{threshold}$$

### Bright-line Emission Spectra

The spectrum of light from a hot gas when passed through a prism was completely different from the well-known rainbow-like pattern from a heated solid, and different gases have different patterns.

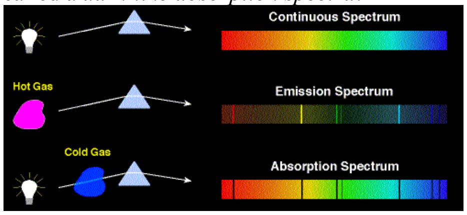
The *bright-line emission spectra* of each element is different, a chemical fingerprint.

Heated Solid



Heated Na vapor

When light from a heated solid is passed through a cool gas, the reverse pattern appears, called a *dark-like absorption spectra*.



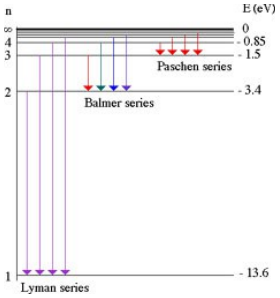
The Swiss mathematics teacher Johann Jakob Balmer published the results of months of work spent manipulating the numerical values of the frequencies of the lines of the visible hydrogen spectrum.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$n$ is an integer and $R = 1.097 \times 10^7\ \text{m}^{-1}$ is the Rydberg constant.

# AP Physics B Review Sheet

Spectral Series predicted for Hydrogen



Lyman series are in the ultraviolet.

Balmer series start as visible light, some UV.

Paschen, Brackett, and Pfund series are in the infrared.

## Bohr's Model (incomplete)

Each possible electron orbit in Bohr's model has a fixed energy called its *energy level*. The fixed energy levels are like rungs in a ladder, but the energy "rungs" are spaced closer together the further you get from the nucleus. The radii "rungs" are further apart as you get further from the nucleus.

Electrons can jump from one energy level to another, but they must gain or lose just the right amount of energy; electrons can't be between energy levels. Electrons can only jump to an orbit where the its angular momentum will increase or decrease by a multiple of $\hbar = h/2\pi$ because angular momentum is quantized.

$$L_n = m v_n r_n = n\left(\frac{h}{2\pi}\right)$$

$n$ is the *principle quantum number* of the electron.

A sudden transition of the electron between two stationary states will produce an emission ($n$ decreases) or absorption ($n$ increases) of radiation with a frequency given by the Planck/Einstein relation

$$hf = E_i - E_f$$

where $E_i$ and $E_f$ are the energies of the atom in the initial and final stationary states. This emission or absorption occurs in a single abrupt step called an *electron transition*.

If the angular momentum of an orbiting body is known, it is a simple matter to compute the radius and the energy of the orbit.

$$r_n = \left(\frac{h^2}{4\pi^2 m_e k e^2}\right)\frac{n^2}{Z}$$

When $n = 1$ and $Z = 1$, the value is 5.3 nm. At this value, called the *Bohr radius*, the energy of the hydrogen atom is a minimum and the atom is said to be in its *ground state*.

$$E_n = -\left(\frac{2\pi^2 m k^2 e^4}{h^2}\right)\frac{Z^2}{n^2}$$

$$E_n = -\left(2.18\times10^{-18}\,\mathrm{J}\right)\frac{Z^2}{n^2}$$

$$E_n = -(13.6\,\mathrm{eV})\frac{Z^2}{n^2}$$

## Quantum Mechanical Model

As in the Bohr model, the *principle quantum number*, *n*, determines the total energy of the atom and determines the size of the *orbital*. It can have only integer values, $n = 1, 2, 3, 4, \ldots$

The *orbital quantum number*, also called the *angular quantum number*, *l*, determines the angular momentum of the electron due to its orbital motion. It determines the different shapes of the orbits. It can only have integer values $l = 0, 1, 2, \ldots, n - 1$.

*s*-shell: $l = 0$
*p*-shell: $l = 1$
*d*-shell: $l = 2$
*f*-shell: $l = 3$

The *magnetic quantum number*, $m_l$, determines the angular momentum of the electron due to its orbital motion. It determines the orientation of the orbital. It can only have integer values, $m_l = -l, \ldots, -2, -1, 0, 1, 2, \ldots, l$

*p*-shell: $p_x$, $p_y$, $p_z$

The *spin quantum number* describes the spin of the electron itself.

*spin up* $m_s = +½$, clockwise rotation
spin down $m_s = -½$, counterclockwise

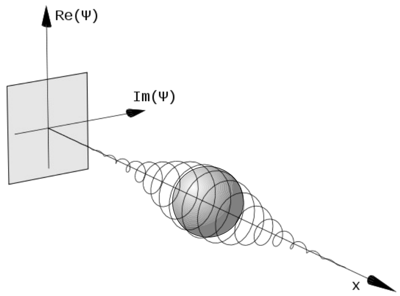
## Pauli Exclusion Principle

Each quantum state, characterized by the four quantum numbers $n$, $l$, $m_l$, and $m_s$, in the atom is limited to one electron. If a state is occupied, the next electron must to to an empty higher energy state, filling up the empty states from the lowest energy to higher energy.
This is what keeps the atom from always collapsing to its lowest or ground state and gives each element its characteristic structure, and that gives the Periodic Table its form.

## Wave-Particle Duality

Young's double-slit experiment proved light behaved as a wave. Einstein's solution to the photoelectric effect proved light behaves as a particle. Light can behave as either, depending on how you measure it.

Prince Louis de Broglie explained this as pilot waves which accompany particles through space and time. He called these waves *pilot*



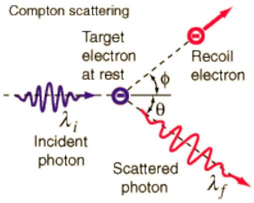
## De Broglie Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

This applies not only to light, but to particles as well.

## The Compton Effect

Arthur Compton used the photon model to explain his research on the scattering of x-rays by the electrons in graphite. The x-ray photon will recoil from the collision in one direction while the electron recoils from the collision in another.



Compton observed that the frequency of the scattered photon is less than the frequency of the incident photon, indicating that the photon loses energy.

He also found that that the difference between the two frequencies depends on the angle at which the scattered photon leaves the collision.

The difference between the wavelength λ' of the scattered photon and the wavelength λ of the incident photon is related to the scattering angle by $\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$

## Heisenberg's Uncertainty Principle

Heisenberg realized that quantum theory implied a fundamental limitation on how accurately certain pairs of physical variables can be measured simultaneously.

He showed from this that there is no way of accurately pinpointing the exact position of a subatomic particle unless you are willing to be quite uncertain about the particle's momentum. Conversely, there is no way of pinpointing the exact momentum of a subatomic particle unless you are willing to be quite uncertain about the particle's position.

$$\Delta x \cdot \Delta p \ge \frac{\hbar}{2}, \text{ where } \hbar = \frac{h}{2\pi}$$

$$\Delta E \cdot \Delta t \ge \frac{\hbar}{2}, \text{ where } \hbar = \frac{h}{2\pi}$$

## Max Born's Matrix Mechanics

The basic idea is that the frequencies of the optical spectrum can be represented as an infinite square matrix, as can the momentum p and displacement q of the oscillators. Then Heisenberg's formula becomes the matrix equation

$$\boldsymbol{pq} - \boldsymbol{qp} = \frac{h}{2\pi i}\boldsymbol{I}$$

where $\boldsymbol{I}$ is the identity matrix.

This leads to a system of equations which could produce the values of the frequencies and relative intensities of spectral lines of atoms. Heisenberg was able to use this matrix formulation to derive all the classical results with his new theory, showing Newtonian mechanics and Maxwell's electromagnetism to be special cases, and deduce the spectra of hydrogen and the additional lines in the presence of magnetic fields.

# AP Physics B Review Sheet

### Erwin Schrödinger's Wave Mechanics

Erwin Schrödinger developed another version based on de Broglie's concept of matter waves. He found an equation which can be applied to any physics system in which the mathematical form of the potential energy *V* is known.

$$\frac{hi}{2\pi}\cdot\frac{\partial^2\Psi}{\partial t^2}=\frac{-h^2}{8\pi^2 m}\nabla^2\Psi+V\Psi$$

$\Psi$ is the wave itself, and is a function of both position and time.

The solution to Schrödinger's equation was a wave that described the quantum aspects of the system. The quantum transitions are now viewed as energy passing continuously from one vibration pattern to another rather than from jumping electrons.
The wave $\Psi$ determines the likelihood that the electron will be in a particular position, but the wave has no physical reality of its own (unlike a sound wave, electromagnetic wave, or water wave). Each point in space around the nucleus has a probability that the electron might be there. The region where the electron is found 90% of the time, according to the wave function solution to the Schrödinger equation, is often called an *electron cloud*.

### Paul Dirac's Transformation Mechanics

At first puzzled by the non-commuting quantities in Schrödinger's wave mechanics, Dirac realized that this was the essence of the new approach. He quickly found a link to classical physics and used the new fundamental idea of non-commutability to develop his own version of quantum mechanics.

Dirac showed that both of the other formulations of quantum mechanics could be viewed as special cases of his own, more general, formulation. In other words, all three, though appearing quite different, are all equivalent.

Dirac also showed that quantum theory had the answer to the apparent paradox of light being both a particle and a wave. The concept of a continuous field was now broken up into bits in order to interact with matter, transforming it into a quantum field. This new approach could treat light as waves or particles, and give the right answers either way. Since this work of Dirac, the dual nature of light as wave and particle has been free of paradox for those who can follow the mathematics.

### Quantum Electrodynamics (QED)

After World War II, Dirac's pioneering work was carried forward by Richard Feynman, Freeman Dyson, Julian Schwinger, and Sin-Itiro Tomonaga. Their quantum electrodynamics theory describes the interaction of light and matter with remarkable accuracy.

## NUCLEAR PHYSICS

The *nucleus* of an atom consists of protons and neutrons, collectively known as *nucleons*.

| | | |
|---|---|---|
| proton | $p^+$ | ${}^1_1H$ |
| neutron | $n^0$ | ${}^1_0n$ |
| electron | $e^-$ | ${}^0_{-1}e$ |

The proton charge is $+1.602\ 177\ 22\times10^{-19}$ C
The electron charge is $-1.602\ 177\ 22\times10^{-19}$ C

The number of protons, *Z*, in the nucleus determines the type of element and is called the *atomic number* of the element.

The number of neutrons, *N*, in the nucleus of an atom can differ between atoms of the same element. The total number of protons and neutrons is called the *atomic mass number* or the *nucleon number*, of that atom, *A*. Atoms of the same element with different atomic mass numbers are called *isotopes*.

$${}^{A}_{Z}X^{charge}_{count}$$

The *atomic mass* of an element is the weighted average of the atomic mass numbers of the different isotopes of that element.

An *atomic mass unit*, amu or u, is defined as exactly one-twelth the mass of a carbon-12 atom.

$$1\text{ amu} = 1.6605\times10^{-27}\text{ kg} = 931.5\text{ MeV}$$

( Note: $1\text{ eV} = 1.6022\times10^{-19}$ J )

A proton has a mass of 1.007 276 u
or $1.672\ 623\ 1\times10^{-27}$ kg
A neutron has a mass of 1.008 665 u
or $1.674\ 928\ 6\times10^{-27}$ kg
An electron has a mass of 0.000 548 579 9 u
or $9.109\ 389\ 7\times10^{-31}$ kg

### Nuclear Size

$$r\approx(1.2\times10^{-15}\,m)\,A^{1/3}$$

### Strong Nuclear Force

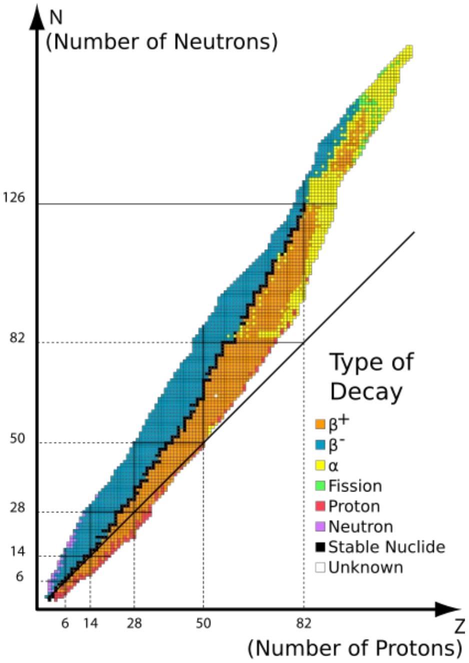
The force that holds the nucleus together is called the *strong nuclear force*. It is about 100 times stronger than the electrostatic force, but its range of action is very short, being very strong at distances less than a femtometer ($10^{-15}$ m) but essentially zero at larger distances. For comparison, the 1s energy level for hydrogen is about 52,918 fm.

The strong nuclear force is almost independent of electric charge; at a given separation distance, nearly the same strong force exists between two protons, between two neutrons, or between a proton and a neutron.

### Nuclear Stability

As the number *Z* of protons in the nucleus increases, the number *N* of neutrons must increase even more in order to keep the nucleus stable.

As more protons are present in a nucleus, there comes a point where adding neutrons still can't hold the nucleus together. Bismuth-209 is the largest stable atom, anything larger is unstable and must break down into smaller atoms via *radioactivity*.



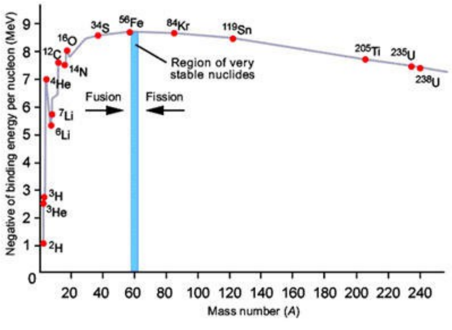
### Binding Energy and Mass Defect

Energy is required to separate a stable nucleus into its constituent protons and neutrons. This energy is called the *binding energy* of the nucleus.

In Einstein's theory of special relativity, mass and energy are equivalent by the famous equation $E = m\,c^2$, where *c* is the speed of light in a vacuum. Thus the binding energy used to disassemble the nucleus appears as extra mass in the separated nucleons.

The difference in mass between the separated nucleons and the stable nucleus is called the *mass defect* of the nucleus.

### Nuclear Stability Again



# AP Physics B Review Sheet

### Radioactivity

*Alpha rays*, α rays, are the least penetrating, being blocked by sheets of lead approximately 0.01 mm thick. They consist of positively charged α particles which are ${}^{4}_{2}He$ nuclei.

*Beta rays*, β rays, penetrate a lead sheet ten times as far, approximately 0.1 mm. They consist of negatively charged $\beta^-$ particles which are electrons or positively charged $\beta^+$ particles which are positrons. These come from the nucleus, not the electron cloud.

*Gamma rays*, γ rays, can pass through a great amount of lead sheeting, approximately 100 mm. They are photons with short wavelengths, high frequencies, and high energies.

### Radioactive Decay

The original nucleus is called the *parent nucleus*. The new nucleus, after the removal of the α particle or β particle, is called the *daughter nucleus*.

Conservation of mass/energy, conservation of linear momentum, conservation of angular momentum, conservation of electric charge, and conservation of nucleon number must be obeyed during radioactive decay.

### Alpha Decay

When the nucleus is too big, or has too many protons, it disintegrates via α decay.

$${}^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}He$$

When a nucleus releases an α particle, energy is also released, due to the change in mass. The energy released appears as the kinetic energy of the daughter nucleus, the kinetic energy of the α particle, and a γ ray.

$$v_d = -\frac{m_\alpha}{m_d} v_\alpha$$

### Beta Decay

When the nucleus contains too many neutrons, it disintegrates via $\beta^-$ decay.

$${}^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e + \bar{\nu}_e$$

In $\beta^-$ decay, the weak interaction converts a neutron *n* into a proton *p* while emitting an electron $e^-$ and an *electron antineutrino*.

When the nucleus has too many protons to be stable, but not enough nucleons to throw out an α particle, it disintegrates via $\beta^+$ decay.

$${}^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + {}^{0}_{1}e + \nu_e$$

In β+ decay, the weak interaction converts a proton *p* into a neutron *n* while emitting an positron $e^+$ and an *electron neutrino*.

A third kind of β decay sometimes occurs when the nucleus pulls in or captures one of the orbital electrons from outside the nucleus. The process is called *electron capture*, or *K capture*, since the electron normally comes from the innermost or K shell (n = 1).

$${}^{A}_{Z}P + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}D + \nu_e$$

### Gamma Decay

The nucleus, like the orbital electrons, exists only in discrete energy states or levels. When a nucleus changes from an excited energy state (denoted by an asterisk *) to a lower energy state, a photon is emitted, like with electrons.

$${}^{A}_{Z}P^* \rightarrow {}^{A}_{Z}P + \gamma$$

### Weak Nuclear Force

The emission of neutrinos and β particles involves a force called the *weak nuclear force*.

The weak force's field strength is $10^{-11}$ times the strength of the electromagnetic force and some $10^{-13}$ times that of the strong force, when forces are compared between particles interacting in more than one way.

### Radioactive Decay

An individual radioactive nucleus will decay randomly. However, given a large sample of a radioactive isotope, the statistical analysis of how many of the radioactive isotopes have decayed follows the mathematical formula:

$$N = N_0 e^{-\lambda t}$$

The *activity, A,* of a radioactive sample is the number of disintegrations per second that occur. The constant λ is called the decay constant.

$$A = \frac{\Delta N}{\Delta t} = -\lambda N$$

The SI unit for activity is the *becquerel*, Bq. It can also be measured in *curie*, Ci.

1 Bq = 1 disintegration/second
1 Ci = $3.70 \times 10^{10}$ Bq

The *half-life* $T_{1/2}$ of a radioactive isotope is the time required for one half of the parent nuclei to disintegrate into daughter nuclei.

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

### Radioactive Dating

The activity of the sample and the original activity can be measured without harming the object, so comparing these is usually the method chosen to date an object.

$$A = A_0 e^{-\lambda t}$$

A more accurate means is to obtain a count of the number of radioactive nuclei present using a mass spectrometer, but this requires removing a portion of the object in most cases.

The best isotope to use is for radioactive dating is one where the half-life of the isotope is neither too short nor too long relative to the age of the sample to be dated.

${}^{210}$Pb – 22.2 years
${}^{14}$C – 5730 years
${}^{238}$U – 4.47×109 years
${}^{40}$K – 1.251×109 years